International Journal of Engineering, Science and Mathematics
Vol.7,Issue 11, November2018,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A

SPLIT DOMINATION IN FUZZY PLANAR GRAPH

K. Priya*
S. Manonmani**

Abstract

Fuzzy graph is now a very important research area due to its wide application. Fuzzy Planar Graph is an important subclass of fuzzy graph. The split domination number of G is the minimum fuzzy cardinality of a split dominating set. In this paper, we introduced a new concept of "Split Domination in Fuzzy Planar Graph". It is combination of spilt domination and fuzzy planar graph. Here, we studied the relation between a split domination number and value of planarity. We derived some results by using minimal domination set.

Keywords:

Fuzzy graph,
Fuzzy planar graph,
Split domination number,
minimal domination set,
Domination in fuzzy planar graph.

Author correspondence:

Assistant Professor*, Department of Mathematics,
Idhaya College for Women. Kumbakonam.
M.Phil Scholar**
Sakthi College of Arts and Science for Womens,
(Affiliated to Mother Teresa Women's University, Kodaikanal.)
Oddanchatram-624619, Dindigul(Dt), Tamilnadu, India.

I. INTRODUCTION

Fuzzy graph theory is one of the developing section in Mathematics. Many authors were derived more results in fuzzy graph theory.

We discussed the concept of dominating graph were introduced by V.R. Kulli and Bidarhalli Janakiraman [5]. We used the Concept Of Fuzzy Planar Graph with using planarity value is introduced by Sovan Samantha, Anita Pal, Madhumangal Pal [8] and Domination in Fuzzy Graph is introduced by A.Somasundaram and S.Somasundaram [6]. Q. M. Mahyoub and N.D.Soner [2] initiate the split dominating set and split dominating number in fuzzy graphs. M. Nithyakalyani and S. Manonmani [4] introducedn the dominating in fuzzy planar graphs. In this paper we introduced split domination in fuzzy planar graph and also some results.

II. PRELIMINARIES

Definition: 2.1

A ***finite graph*** is a graph $G = (V, E)$ such that V and E are called vertices and edges finite sets.

Definition: 2.2

An ***infinite graph*** is one with an infinite set or edges or both. Most commonly in graph theory, it is implied that the graphs discussed are finite.

Definition: 2.3

If more than one edge joining two vertices is allowed, the resulting object is a ***multigraph***[1]. Edges joining the same vertices are called ***multiple lines***.

Definition: 2.4

A drawing of a geometric representation of a graph on any surface such that no edges intersect is called ***embedding***.

Definition: 2.5

A graph G is ***planar*** [1] if it can be drawn in the plane with its edges only intersecting at vertices of G. So the graph is ***non-planar*** if it cannot be drawn without crossings.

Definition: 2.6

A $\boldsymbol{fuzzy\ set\ A}$ [3] on a universal set X is characterized by a mapping $m: X \to [0,1]$, which is called the membership function. A fuzzy set is denoted by $A = (X, m)$.

Definition: 2.7

A $\boldsymbol{fuzzy\ graph}$ [3] $G = (V, \sigma, \mu)$ is a non-empty set V together with a pair of function $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$ such that for all $x, y \in V$, $\mu(x, y) \leq \sigma(x) \wedge \sigma(y)$ Where $\sigma(x)$ and $\mu(x, y)$ represent the membership values of the vertex x and of the edge (x, y) in G respectively.

Definition: 2.8

The fuzzy graph $G - (V, \sigma, \mu)$, an edge (x, y) is called $\boldsymbol{strong}$ [7] if $\frac{1}{2}\{\sigma(x) \wedge \sigma(y)\} \leq \mu(x, y)$ and weak otherwise.

Definition: 2.9

The ***order p*** [4] and ***size q*** [6] of a fuzzy graph $G = (\sigma, \mu)$ are defined to be $p = \sum_{x \in V} \sigma(x)$ and $q = \sum_{x,y \in E} \mu(xy)$.

Definition: 2.10

$N(x) = \{y \in V \mid \mu(xy) = \sigma(x) \wedge \sigma(y)\}$ is called the ***neighborhood of x*** and $N[x] = N(x) \cup \{x\}$ is called the ***closed neighborhood of x*** [6].

Definition: 2.11

If an edge (x, y) of a fuzzy graph satisfies the condition $\mu(x, y) = \sigma(x) \wedge \sigma(y)$, then this edge is called ***effective edge*** [8].

Definition: 2.12

Two vertices are said to be ***effective adjacent*** [8] if they are the end vertices of the same effective edge.

Definition: 2.13

The ***minimum neighborhood degree*** is denoted by δ_N and ***maximum neighborhood degree*** is denoted by Δ_N

Definition: 2.14

Let *G be a* ***fuzzy planar graph*** with planarity value f**,** [8] where

$f = \frac{1}{1+\{I_{P_1}+I_{P_2}+\cdots+I_{P_n}\}}$. The range of f is $0 < f \leq 1$.

Here, $P_1, P_2, \ldots . P_n$ be the points intersections between the edges.

In a graph $G = (V, \sigma, E)$, E contains two edges $\mu(a, b)$ and $\mu(c, d)$, which are intersected at a point P.

Strength of the fuzzy edge $I_{(a,b)} = \frac{\mu(a,b)}{\{\sigma(a)\wedge\sigma(b)\}}$.

The intersecting point at P is $I_P = \frac{I_{(a,b)}+I_{(c,d)}}{2}$.

Results:

If there is **no point of intersection** for a geometrical representation of a fuzzy planar graph, then **its fuzzy planarity value is 1**.

- If $\mu(w, x) = 1$ (or near to 1) and $\mu(y, z) = 0$ (near to 0), then we say that the fuzzy graph has no crossing. Then the crossing will not be important for planarity.
- If $\mu(w, x) = 1$ (or near to 1) and $\mu(y, z) = 1$ (near to 1), then the crossing will be important for planarity.
    - Strong fuzzy planar graph if f is greater than or equal 0.5.
    - Otherwise weak.

Definition: 2.15

Let $G = (V, E)$ be a graph. A set $D \subseteq V$ is a ***Dominating Set*** [5] of G if every vertex in $V \backslash D$ is adjacent to some vertex in D.

Definition: 2.16

The ***minating set*** $\boldsymbol{\gamma(G)}$ [5] of G is the minimum cardinality of a dominating set.

Definition: 2.17

A dominating set D is a ***minimal dominating set*** [5] if no proper subset $D' \subset D$ is a dominating set of G.

Definition: 2.18

Let $G = (V, \sigma, \mu)$ be a fuzzy graph on V. Let $x, y \in V$, if x dominates y in G if $\mu(x, y) = \sigma(x)\wedge\sigma(y)$. A subset D of V is called a ***dominating set*** [6] in G if for every $v \notin D$, there exists $u \in D$ such that u dominates v.

Definition: 2.19

The minimum fuzzy cardinality of a dominating set in G is called the ***dominating number*** [6] of G and is denoted by $\zeta(G)$ or ζ.

Definition: 2.20

Let G be a fuzzy graph without isolated vertices. A subset D of V is said to be a ***Total Dominating set*** [6] if every vertex in V is dominated by a vertex in D.

The ***Total Domination Number*** of G is denoted by ζ_t.

Definition: 2.21

If a graph G is said to be ***Domination in Fuzzy Planar Graph*** [4] if

- $G = (V, \sigma, \mu)$ be a fuzzy planar graph with planarity value *f*.
- Let $x, y \in V$, x dominates y in G then $\mu(x, y) \leq \sigma(x)\wedge\sigma(y)$.
- A subset D of V is called a dominating set in G if for every $y \notin D$, there exist $x \in D$ such that x dominates y.
- The minimum fuzzy cardinality of a dominating set in G is called the dominating number of G and is denoted by ζ_{FP}.

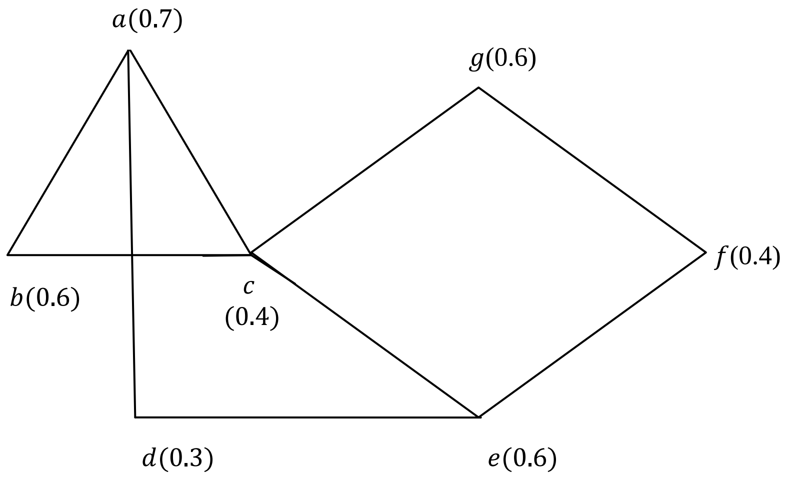
Definition: 2.22

A dominating set D of a fuzzy graph $G = (\mu, \rho)$ is a **Split Dominating** [2] set if the induced subgraph $H = (< V - D >, \mu', \rho')$ is disconnected. The minimum fuzzy cardinality of a split dominating set is called a split domination number and is denoted by $\zeta_s(G)$.

III. SPLIT DOMINATION IN FUZZY PLANAR GRAPH

Definition: 3.1

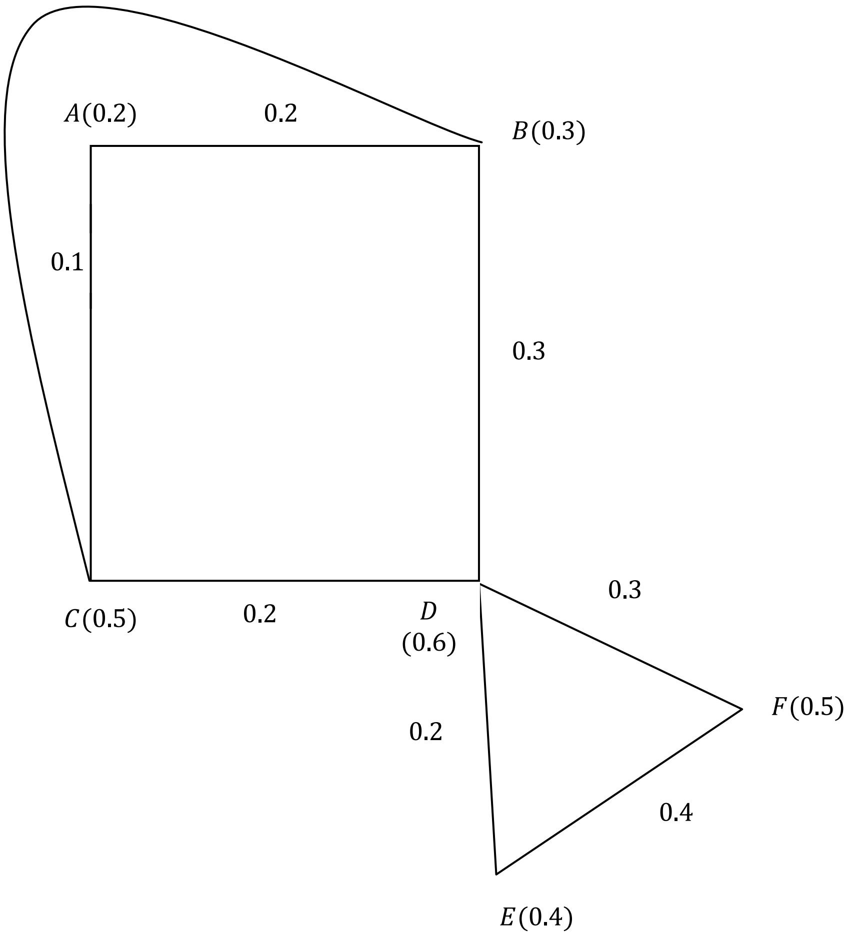
A domination set D of a fuzzy planar graph with planarity value f is **a Split Domination Set** if the induced fuzzy planar subgraph $V - D$ is disconnected.
Domination set is denoted by D_{SPFP}. Minimum domination number is denoted by ζ_{SPFP}.
Example:3.2



$D_{SPFP} = \{c, d, g\}$, $\zeta_{SPFP} = 3$.

Definition: 3.3

A dominating set D of a Fuzzy Planar Graph G is said to be a **Minimal Split Domination Set** if no proper subset of D.
Theorem : 3.4
Let G be a fuzzy planar graph the planirity value f and ζ_{SPFP} be a Minimum split domination number then $f \leq \zeta_{SPFP}$.
Proof:
Let G be a fuzzy planar graph with the planarity value f. Where f is bounded and $0 < f \leq 1$.
Let ζ_{SPFP} be a minimum split domination number of fuzzy planar graph.
Here, possibility of minimum split domination number should be 1. Hence $f \leq \zeta_{SPFP}$.



$D_{SPFP} = \{D\}$, $f = 0.571$, $\zeta_{SPFP} = 1$, $f \leq \zeta_{SPFP}$, $0.57 \leq 1$.
Hence proved.

Theorem : 3.5

A split dominating set D_{SPFP} of a fuzzy planar graph G is a minimal split dominating set iff for each $x \in D_{SPFP}$. Then it should hold any one of the following

i. $N(x) \cap D_{SPFP} = \emptyset$.
ii. There is a vertex.

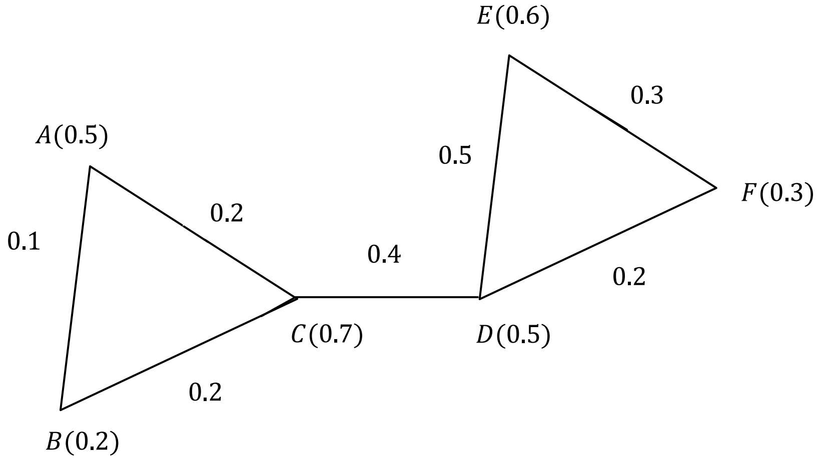
$y \in v/D_{SPFP}$ such that $N(y) \cap D_{SPFP} = \{x\}$

Proof:

Let D_{SPFP} be a minimum split dominating set.

$x \in D_{SPFP}$ then $D_o = D_{SPFP} - \{x\}$ is not a split dominating set and hence there exist $a \in V - D_o$ suchthat a is not dominated by any element of D_o .
If $a = x$ we will get i. If $a \neq x$ we will get ii.

International Journal of Engineering, Science and Mathematics
Vol.7,Issue 11, November2018,
ISSN: 2320-0294 Impact Factor: 6.765
Journal Homepage: http://www.ijesm.co.in, Email: ijesmj@gmail.com
Double-Blind Peer Reviewed Refereed Open Access International Journal - Included in the International Serial Directories Indexed & Listed at: Ulrich's Periodicals Directory ©, U.S.A., Open J-Gage as well as in Cabell's Directories of Publishing Opportunities, U.S.A



$V = \{A, B, C, D, E, F\},\ D_{SPFP} = \{C, D\}$.
$C \in D_{SPFP} \Longrightarrow N(C) = \{A, B, D\}, D_o = \{C, D\} - \{C\} = \{D\}$.
$V - D_o = \{A, B, C, D, E, F\} - \{D\} = \{A, B, C, E, F\}$.
If $D = C,\ N(C) \cap D_{SPFP} = \{A, B, D\} \cap \{C, C\} = \emptyset$
$a \in V - D_{SPFP},\ N(a) \cap D_{SPFP} = \{C\}$.
Hence proved.

Theorem: 3.6

Let G be a fuzzy planar graph without isolated vertices and D_{SPFP} be a minimum split dominating set of G. Then $V - D_{SPFP}$ is also be a split dominating set of G.

Proof:

Let $y \in D_{SPFP}$. Since G has no isolated vertices there is a vertex $x \in N(y)$. From the above theorem,
$y \in V - D$, thus every element of D is dominated by some element of $V - D$.
Hence proved.

Theorem :3.7

A dominating set D of a fuzzy planar graph G is a split dominating set D_{SPFP} iff there exist 2 vertices $u, v \in V - D$ such that every $u - v$ contains a fuzzy vertex of D.

Proof:

Let D be a split dominating set D of a fuzzy planar graph G. Then the induced subgraph $< V - D >$ is disconnected.

Hence there exist two vertices $u, v \in V - D$ such that every $u - v$ path contains a fuzzy vertex of D. be a dominating set then the induced subgraph $V - D$ is connected or disconnected.

Suppose if it is connected then there exist $u, v \in V - D$ such that $u - v$ path atleast not contains fuzzy vertex of D. which is contradiction.

Hence $V - D$ is disconnected which implies D is a split dominating set of a fuzzy planar graph G. Hence proved.

IV. CONCLUSION

The concept of "Split Domination in Fuzzy Graph" has been discussed in this paper. We derived the results by using split domination in fuzzy planar graph and minimal dominating set

REFERENCES

[1] S. Arumugam and S. Ramachandran, "Invitation to graph theory" Scitech Publications (India) Pvt. Ltd., (2012).

[2] Q. M. Mahyoub and N. D. Soner, "The Split Domintion Number Of Fuzzy Graphs", Far East Journal of Applied Mathematics , 31-1, (2008), pp. 125-132.

[3] A. Nagoorgani and V.T. Chandrasekaran, "A First Look At Fuzzy Graph Theory", Allied Publication Pvt. Ltd.

[4] M. Nithya Kalyani and S. Manonmani, Domination In Fuzzy Planar Graph, International Journal of Engineering, Science and Mathematics, 7-3, (2018).

[5] V. R. Kulli and Bidarhalli Janakiram, "The dominating graph," Graph Theory Notes of New York XLVI, 5-8 (2004).

[6] A. Somasundaram and S. Somasundaram "Domination in fuzzy graph - I", Pattern Recognition Letters, 19, (1998), pp.787-791.

[7] S. Ismail Mohideen and A. Mohamed Ismayil "Domination in Fuzzy Graph: A New Approach" , International journal of Computational Science and Mathematics, 2- 3, (2010), pp. 101-107.

[8] S. Samanta , M. Pal and A. Pal, "New concept of fuzzy planar graph", International Journal of Advanced Reseach in Artificial Intelligence, 3-1, (20014), pp. 52-59.